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# The variable Thickness of Thermal Gradient of an Elastic Plate

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**ABSTRACT-**

*The dynamic analysis of plates with non-linear thickness variation and thermal gradient has been of interest to structural analysis and designers for several decades. Analysis techniques for plates are strongly dependent on their boundary conditions and geometrical shap. The plates with supports play an important role in engineering field, such as slabs, on columns in civil engineering and printed circuit board in electronic engineering. Due to their practical importance, the free vibration analysis of the plates has received considerable attention.*

**Key Words**: Vibrations, Oscillations, centrifugal, exponentially.

## Introduction-

Vibration refers to mechanical oscillations about an equilibrium point. The oscillations may be periodic such as the motion of a pendulum or random such as the movement of a tire on a gravel road. Early, Scholars in the field of vibrations concentrated their efforts on understanding the natural phenomenon and developing mathematical theories to describe the vibration of physical system. In recent times, many investigations have been motivated by the engineering application of vibration, such as the design of machine, foundations, strutures, engines, turbines and control system. Most prime movers have vibration problem due to the inherent imbalance in the engines. The imbalance may be due to faulty design or poor manufacture. Imbalance in diesel engines can cause ground waves sufficiently powerful to create a nuisance in urban areas. In turbines, vibration cause spectacular mechanical failures. Engineers have not yet been able to prevent the failures that result from plate and due vibrations in turbines. Naturally the structures designed to support heavy centrifugal machines, like motors and turbines or reciprocating machines like steam and gas engines and reciprocating pumps are also subjected to vibration.

Vibrations of rectangular plate of non-linear varying thickness in the presence of thermal gradient have been studied. Thermal effect is considered as linear in x-direction only. Spline interpolation technique has been applied to determine the frequency equation of the plate. The plate is simply supported along two opposite sides and is clamped/simply supported along the other sides. The frequencies corresponding to the first three modes of vibration are obtained for a rectangular plate for different values of thermal gradient, taper constants and aspect ratio. Numerical results are shown in tabular as well as graphically form.

The influence of thermal field is harmonically along x axis. Spline interpolation technique has been applied to determine the frequency equation of the plate. The plate is simply supported along two opposite sides and is clamped/simply supported along the other sides. The frequencies corresponding to the first three modes of vibration are obtained for a rectangular plate for different values of thermal gradient, taper constants and aspect ratio. Numerical results are shown in tabular as well as graphically form.

Vibrations of rectangular plate of non-linear varying thickness in the presence of exponentially thermal gradient have been studied. Thermal effect is considered as exponentially in x-direction only. Spline interpolation technique has been applied to determine the frequency equation of the plate. The plate is simply supported along two opposite sides and is clamped/simply supported along the other sides. The frequencies corresponding to the first three modes of vibration are obtained for a rectangular plate for different values of thermal gradient, taper constants and aspect ratio. Numerical results are shown in tabular as well as graphically form.

Thermal effect is considered as linear in x-direction only. Thickness variation is considered as non-linear in x-direction and linear in y-direction. Raylegh-Ritz technique has been applied to determine the frequency equation of the plate. The plate is clamped along all the four siders. The frequencies corresponding to the first two modes of vibration are obtained for a rectangular plate for different values of thermal gradient taper constants and aspect ratio. Numerical results are shown in tabular as well as graphically form.

**ASSUMPTIONS-**

Vibrations of rectangular plate of non- linear varying thickness in the presence of thermal gradient have been studied. Thermal effect is considered as linear in x-direction only. Spline interpolation technique has been applied to determine the frequency equation of the plate. The plate is simply supported along two opposite sides and is clamped / simply supported along the other sides. The frequencies corresponding to the first three modes of vibrations are obtained for a rectangular plate for different of thermal gradient, taper constants and aspect ratio. Numerical results are shown in tabular is well as graphically form.

Harmonically thermally induced vibration of a rectangular plate with thickness varies non-linear in x –direction is studied. The influence of thermal field is harmonically along x axis. Spline interpolation technique has been applied to determine the frequency equation of the plate. The plate is simply supported along two opposite sides and clamped / simply supported along the other sides. The Frequencies corresponding to the first three modes of vibrations are obtained for a rectangular plate for different values of thermal gradient, taper constants and aspect ratio. Numerical results are shown in tabular as graphically form.

Vibrations of rectangular plate of non- liner varying thickness in the presence of exponentially thermal gradient have been studied. Thermal effect is considered as exponentially in x- directions only. Spline interpolation technique has been applied to determine the frequency equations of the plate. The plate is simply supported along two opposite sides is clamped simply supported along the other sides. The frequencies corresponding to the first three modes of vibrations are obtained for a rectangular plate for different values of thermal gradient, taper constants and aspect ratio. Numerical results are show in tabular as well as graphically form.

Let f(X) be a function with continuous derivatives in the range (0.1). Choose (n+1) points $X_0$, $X_1$, $X_2$, . .., $X_n$ , in the range $0 \le X \le 1$ such that $0 = X_0 < X_1 < X_3, ..., < X_n = 1$.

Let the approximating function $W(X)$ for $f(X)$ be a quantic spline with the following properties:

a) $W(X)$ is a quantic polynomial in each interval $(X_k, X_{k+1})$,

b) $W(X_k) = f(X_k), k = 0(1)n,$

c) $W'(X), W''(X), W'''(X)\ and\ W^{iv}(X)$ and continuous.

From the definition, a quantic spline takes the form

$$W(X) = a_0 + \sum_{i=1}^{4} a_i (X - X_0)^i + \sum_{j=0}^{n-1} b_j (X + X_j)^5 + \quad (3.13)$$

where

$$(X - X_j)_+ = \begin{cases} 0\ if\ X < XJ \\ x - xj\ if\ x \ge X_J \end{cases} \quad (2.14)$$

It is also assumed, for simplicity, that the knots $X_i$ are equally spaced in (0, 1) with the spacing interval $\Delta X$, so that

$$\Delta X = 1/n,$$

$$X_i = i\Delta X (i = 0, 1, 2, \ldots n). \quad (3.15)$$

The number of unknown constants in equation (3.13) is (n+5). Satisfaction of differential equation (3.11) by collocation at the (n+1) knots in the interval (0, 1) together with the boundary conditions (to explained in the next section) gives precisely the requisite number of equations for the determination of unknown constants.

Substituting $W(X)$ from equation (3.13) into equation (3.11) gives, for satisfaction at the m[th] knot, one obtains

$$B_4 a_0 + \left[B_4\left(X_q - X_0\right) + B_3\right]a_1 + \left[B_4\left(X_q - X_0\right)^2 + 2B_3\left(X_q - X_0\right) + 2B_2\right]a_2 +$$
$$\left[B_4\left(X_q - X_0\right)^3 + 3B_3\left(X_q - X_0\right)^2 + 6B_2\left(X_q - X_0\right) + 6B_1\right]a_3 +$$
$$\left[B_4\left(X_q - X_0\right)^4 + 4B_3\left(X_q - X_0\right)^3 + 12B_2\left(X_q - X_0\right)^2 + 24B_1\left(X_q - X_0\right) + 24B_0\right]a_4 +$$
$$\sum_{i=0}^{n-1}\left[B_4\left(X_q - X_i\right)^5 + 5B_3\left(X_q - X_i\right)^4 + 20B_2\left(X_q - X_i\right)^3 + 60B_1\left(X_q - X_i\right)^2 + 120B_0\left(X_q - X_i\right)\right]b_i = 0$$

(3.16)

Where

$$B_0 = \left[1 - \alpha(e - e^{xq})/(e-1)\right]\left(1 + \beta_1 X_q + \beta_2 X_q^{\ 2}\right)^2,$$

$$B_1 = 2[\alpha e^{xq}/(e-)\left(1 + \beta_1 X_q + \beta_2 X_q^{\ 2}\right)^2 + 3(1-\alpha(e-e^{xq})(e-1)$$
$$(1+\beta_1 X_q + \beta_2 X_q^{\ 2})(\beta_1+2\beta_2 X_q)],$$

$$B_2 = [(\alpha e^{xq}/(e-1))\left(1 + \beta_1 X_q + \beta_2 X_q^{\ 2}\right)^2 + 6(\alpha e^{xq}/(e-1))$$
$$(1+\beta_1 X_q + \beta_2 X_q^{\ 2})(\beta_1+2\beta_2 X_q)+6(1-\alpha(e-e^{xq})/(e-1))(\beta_1+2\beta_2 X_q)^2$$
$$+6(1-\alpha(e-e^{xq})/(e-1))\ (1+\beta_1 X_q + \beta_2 X_q^{\ 2})\beta_2)-2r^2$$
$$(1-\alpha(e-e^{xq})/(e-1))(1+\beta_1 X_q+\beta_2 X_q^{\ 2})^2],$$

$$B_3 = -2r^2[(\alpha e^{xq}/(e-1))\left(1 + \beta_1 X_q + \beta_2 X_q^{\ 2}\right)^2 + 3(1-\alpha(e-e^{xq})/(e-1))$$
$$(1+\beta_1 X_q + \beta_2 X_q^{\ 2})(\beta_1+2\beta_2 X_q),$$

$B_4 = r^2[\mathrm{r}^2(1\text{-}\alpha(\mathrm{e}\text{-}e^{xq})/(\mathrm{e}\text{-}1))\left(1+\beta_1 X_q+\beta_2 X_q{}^2\right)^2 - \nu((\alpha e^{xq}/(\mathrm{e}\text{-}1))$

$(1+\beta_1 X_q+\beta_2 X_q{}^2)+6(\alpha e^{xq}/(\mathrm{e}\text{-}1))(1+\beta_1 X_q+\beta_2 X_q{}^2)(\beta_1+2\beta_2 \mathrm{X}_q)$

$+6(1\text{-}\alpha\cos(\pi/2)\mathrm{X}_q)(\beta_1+2\beta_2\mathrm{X}_q)^2+6(1\text{-}\alpha(\mathrm{e}\text{-}e^{xq})/(\mathrm{e}\text{-}1))$

$(1+\beta_1 X_q+\beta_2 X_q{}^2)+6(1\text{-}\alpha(\mathrm{e}\text{-}\mathrm{e}^{\mathrm{X}_q})/(\mathrm{e}\text{-}1)(1+\beta_1 X_q+\beta_2 X_q{}^2)\beta_2)]\text{-}\lambda^2.$

Thus, one obtains a homogeneous set of equations in terms of unknown constants $a_0, a_1, a_2, a_3, a_4, b_0, b_1 \,..........b_{n-1}$, which, when written in matrix notation, takes the form

$$[B][C]=0 \qquad (3.17)$$

where $[B]$ is an $(n+1)\times(n+5)$ matrix and $[C]$ is an $(n+5)\times 1$ matrix.

**Research Methodology-**

As no work available on non-linear thickness variation on exponentially thermally induced vibration of rectangular plate, so here thermal effect on vibration of rectangular plate of non-linear varying thickness is studied. Here, vibration of a rectangular plate of non-linear varying thickness under a steady exponentially temperature distribution have been studied. The effect of temperature on modulus of elasticity is assumed to vary exponentially along x-axis. The non-linear thickness variation is taken as combination of linear and parabolically variation factor. The differential equation of motion has been solved by quantic spline interpolation technique. The two edges parallel to x-axis (y=0 and y=b) are assumed to be simply supported. Different set of boundary conditions have been imposed at the other two edges. The frequency parameters for the first three mods of vibrations for C-S-C-S- and S-S-S-S-boundary conditions and for various values of taper constants, thermal constant and fixed value of length to breadth ratio are obtained. Results are presented in tabular and graphically form both.

**Results & Findings-**

Frequency equations are transcendental equations in $\lambda^2$ from which infinitely many roots can be obtained. The frequency parameter $\lambda$ corresponding to first three modes of vibration of C-S-C-S- and S-S-S-S- rectangular plates have been computed for m=1 and various values of aspect ratio (a/b), thermal constant $(\alpha)$ and taper constants $(\beta_1, \beta_2)$. The value of Poisson ratio $\nu$ has been taken as 0.3.

To choose the appropriate interpolation interval $\Delta X$, the computer programme has been developed for the evaluation of the frequency parameter $\lambda$ and run for n=10(5)60. The numerical values show a consistent improvement with the increase of the number of knots. In all the above computation, authors have fixed n=50, since further increase in n does not improve the results except in the fifth or sixth decimal places. These results have been tabulated in and have been explained with the help of graphs, plotted between various parameters.

The variation of frequency parameter ($\lambda$) with thermal constant $(\alpha)$ for different combinations or taper constant $(\beta_1, \beta_2)$ and fixed aspect ratio (a/b=1.5) corresponding the first three modes of vibration for C-S-C-S and S-S-S-S plates. The value of frequency parameter decrease with the increase of thermal constant for both the boundary conditions, considered here. Further, it can be seen from table that frequency parameter, for both the boundary conditions. Decreases gradually in the third mode of vibrations in comparison to first two modes of vibration.

The results presented in tables show a marked effect of variation of taper constant $(\beta_1)$ on the frequency parameter for taper constant $(\beta_2 = -0.5, 0.5)$, two values of thermal constant $(\alpha = 0.0, 0.4)$ and fixed aspect ratio (a/b=1.5) corresponding to the first three modes of vibration. It is observed that the frequency parameter increases with the increase of taper constant for both the boundary conditions, considered here. Further, it can be seen frequency parameter increase more sharp for $(\beta_2 = 0.5)$ in comparison of $(\beta_2 = -0.5)$ for both the boundary conditions and all the three modes.

The effect of taper constant $(\beta_2)$ on frequency parameter for taper constant $(\beta_1 = -0.5, 0.5)$, two values of thermal constant $(\alpha = 0.0, 0.4)$ and fixed aspect ratio (a/b=1.5) corresponding to the first three modes of vibration for C-S-C-S- and S-S-S-S- plates, have been shown from this table, one can observe that frequency parameter in fisrt three mods of vibration increases with the increase of taper constant for C-S-C-S- and S-S-S-S- plates. These results are plotted in figures.

The variation of frequency parameter $(\lambda)$ with aspect ratio (a/b) for two values of thermal constant $(\alpha = 0.0, 0.4)$ and two combinations of taper constant $(\beta_1 = \beta_2 = -0.5 \, and \, \beta_1 = \beta_2 = 0.5)$ corresponding the first three modes of vibration for C-S-C-S- and S-S-S- plates. The value of frequency parameter increase with the increase of aspect ratio for both the boundary conditions, considered here. These results are plotted also, one can observe from tables 3.1 to 3.7, that frequency parameter of C-S-C-S- plate is higher than that of S-S-S- plate.

It is evident from the increasing the values of aspect ratio a/b, frequency parameter increases.

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It can be concluded from the results that frequency parameter increases with increase in taper constants and decreases with increase in thermal gradient. Also, it is evident when $\beta_1 = 0.5$, the values of frequency parameter more in comparison to $\beta_2 = 0.5$ for all the three mode of vibrations and both the boundary conditions. It is also clear from the tables that third mode of vibration changes more sharply than second and first.

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